International Journal of Theoretical Physics, Vol. 33, No. 9, 1994

An Alternative Form for Raychaudhuri's Equation

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Received June 16, 1993

Raychaudhuri's equation contains some features that deserve attention, such as the increase with t^2 of the difference between half the density parameter and the deceleration parameter. We derive a form of the equation that shows how this happens.

The Raychaudhuri equation (Raychaudhuri, 1980) is given by

$$\dot{\theta} + \frac{\theta^2}{3} - \dot{u}^k_{;k} + 2(\sigma^2 - w^2) + \frac{1}{2}k(\rho + 3p) - \Lambda = 0 \tag{1}$$

where σ stands for the shear tensor magnitude, $\dot{u}^k_{;k}$ the acceleration, w the spin vector magnitude, ρ the energy density, p the cosmic pressure, Λ the cosmological constant, and θ the dilatation. If we define a function R by

$$H = \frac{\dot{R}}{R} = \frac{\theta}{3} \tag{2}$$

it can be shown that R stands for the average "radius" of the universe.

(For instance, in the Bianchi I metric,

$$ds^2 = dt^2 - A^2\,dx - B^2\,dy - C^2\,dz \tag{3}$$

we would have

$$R^6 = A^2B^2C^2) \tag{4}$$

Then, we can obtain, from (2),

$$\frac{3\ddot{R}}{R} = 2w^2 - 2\sigma^2 + \dot{\mu}^k_{;k} - \frac{1}{2}k(\rho + 3p) + \Lambda \tag{5}$$

(see, for instance, Narlikar, 1983).

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0020-7748/94/0900-1929$07.00/0

Here, H stands for Hubble's parameter, and we get

$$3H^2\left(\frac{r}{2} - q\right) = 2w^2 - 2\sigma^2 + \dot{\mu}^k_{;k} - \frac{3}{2}kp + \Lambda \tag{6}$$

where

$$r = \frac{\rho}{\rho_{\text{crit}}} \tag{7}$$

and

$$q = -\frac{\ddot{R}R}{\dot{R}^2} \tag{8}$$

is the deceleration parameter.

Formula (6) is the desired form for Raychaudhuri's equation in terms of the parameters of the theory. Apparently, the literature on the subject does not mention this alternative formulation (6) for Raychaudhuri's equation.

Relation (6) tells us that the effects of shear, spin, acceleration, pressure, and cosmological constant may be magnified with the age of the universe. In fact, we can very generally write

$$H \cong \frac{1}{(1+q)t} \tag{9}$$

so that

$$3\left(\frac{r}{2} - q\right) = (2w^2 - 2\sigma^2 + \dot{\mu}^k_{;k} - \frac{3}{2}kp + \Lambda)t^2(1+q) \tag{10}$$

For the present universe, we have, hopefully,

$$q = \frac{1}{2} \tag{11}$$

so that

$$R \propto t^{2/3} \tag{12}$$

and

$$p \cong 0 \tag{13}$$

We are left with

$$r - 1 \cong (2w^2 - 2\sigma^2 + \dot{\mu}^k_{;k} + \Lambda)t^2 \tag{14}$$

The *anzatz* $r = 1$ really means that

$$2\sigma^2 = 2w^2 + \dot{\mu}^k_{;k} + \Lambda \tag{15}$$

In conclusion, we stress that the increase in time (with t^2) of the difference $r/2-q$ is something to be considered in the development of cosmological models. If the left-hand side of (10) is constant, then we might think of the following possible solution:

$$w = At^{-1} \qquad (A = \text{const})$$

$$\sigma = Bt^{-1} \qquad (B = \text{const})$$

$$\dot{\mu}^{k}_{;k} = Ct^{-2} \qquad (C = \text{const})$$

$$\Lambda = Dt^{-2} \qquad (D = \text{const})$$

$$p = Et^{-2} \qquad (E = \text{const})$$

Then,

$$3\left(\frac{r}{2} - q\right) = \left(2A^2 - 2B^2 + C - \frac{3}{2}kE + D\right)(1 + q)$$

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